FOUNDATION OF RISK CONCEPT

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ABSTRACT

Concept of risk starts when human species recognize the possibility of losing value after they create or recognize it. We call the event accident if this possibility is realized. Risk should be defined as the product of the total loss caused by the accident and the probability of occurrence of the accident or as the expectation of loss in short. In the deep consideration we need to change the definition of probability from frequency concept to degree of belief concept. By measuring the total loss with the monetary unit and by adopting the degree of belief definition for the probability, risk is assured to be an additive entity, which we can evaluate quantitatively and rationally. We need to recognize that the amounts of value, probability and also risk are attribute to human mind.

1. PREFACE

Observing human activities, it seems to me that human species create, acquire, and maintain values or at least try to do it. As values are recognized by human species, they are fairly changeable to times and places or to people. Even so, human kinds have common sense of values.

As soon as people recognize values they have a fear of not acquiring them or losing them. The reason why they have such a fear is that every event is uncertain to happen whatever the degree of certainty is. This fear is the onset of risk concept.

To allocate limited resources effectively, more rational activities are required in the space development field than ever. To this end we need to evaluate risk properly. However a few confusions not to be overlooked are observed for the time being probably because the word, “risk” has been used in so various fields [1].

It is necessary for us to recognize again that risk is intrinsically an attribute to human mind and to define the concept accurately not vaguely. By doing so, we can assure the foundation for Quantitative Risk Analysis (QRA), and we can make right expression for safety, which is closely relating to risk.

2. LOSS OF THE ACCIDENT AND ITS ESTIMATION

The contents people recognize valuable ranges in vast. It is not limited to the actual existence such as diamond or land. Something we develop now may be also valuable. We proceed to develop it because we indeed recognize its value.

However, values are not fixed as a character of materials even if they are diamond or gold. Values are decided by people and therefore are fluid and vary to person to person. Then dealing comes into existence and economic activities start in the world. Even we know this fact, we can say human kinds have common sense of values in perspective. That is to say, what is precious to someone is also precious to others.

Through the long history of human kind, people have established unique monetary unit for their own country. Generally speaking, value is expressed with monetary unit. However, there are many people who deny that all of value can be measured with monetary unit. They claim life of human is invaluable literally. Even so, it is certain we have to negotiate with monetary unit in the situation where we ask the compensation for the traffic accident incurred human life. Allow us the postulate saying all of value can be measured with or converted to monetary value.

In this paper an accident is the event we identified as a matter of issue and unfavorable if it really occurred. The magnitude of value we lose when the accident occurred can be estimated with monetary unit, not asking the accuracy of the estimation. The values we lose are not only material but also mental, and in addition it is natural that the evaluation is fairy different among evaluators. Therefore we should be satisfied as accurate enough estimation if we could show the order of value measured with monetary unit. Surprisingly however we have used only four kinds of qualitative words, catastrophic, critical, marginal, and negligible for the expression of the amount of loss in the risk evaluation. These are too less expressions for the quantitative evaluation even the case of evaluation limited to inside of a program because there is no additivity in the expression. Therefore, we need to estimate the amount of loss with monetary unit.

Because monetary unit is so established as the additive rule are satisfied, it is apparent the additive rule are also satisfied for the amount of loss, as long as it is estimated with monetary unit.

3. PROBABILITY OF OCCURRENCE

People who are not Gods cannot have 100% degree of belief on the occurrence of any event whichever it is natural or artificial. We cannot predict the typhoon coming or earthquake occurrence with 100% degree of belief. We could not expect absolutely success for even simple operation such as the bolt cutter we made. We have to expect some possibility of failure not to mention in the advanced technical area such as launching satellite. What we can do is limited to show with any method how we reduced the degree of failure possibility depending on the property of the event.

In the old days, quantitative probability expressions were used for the degree of possibility on the occurrence of the accident in the evaluation of risk (for example, [2]). Nowadays, qualitative expression governs for risk evaluation. For example, these are “frequent”, “reasonably probable”, “occasional”, “remote”, “extremely impossible”, and “no possibility”. We should notice these expressions are too subjective and the impressions of these words vary from person to person.

We need to remind the fact that the additivity is required for risk evaluation. Qualitative expression for probability cannot satisfy this requirement. Therefore, we have to go back to the numerical expression, for example, “frequent” means one in ten, or “no possibility” means less than once a millionth, that is probability.

I presume two reasons why this probability expression was refused. First one is a psychological operation, that is, even we are willing to say “frequent” but we hesitate to say explicitly “with the probability of around 1/10”. I suppose for the reason that we are not accustomed to the slightly small probability except the probability of casting die. This may be same situation that we do not have intuition for very large number such as billion or trillion without a training. We could have a better sense of feeling for probability also by training.

I presume another reason for avoidance of probability expression. This is derived from the definition of probability adopted in the reliability engineering. This definition has been adopted for seeking objectivity to the probability. We need to consider again that we can take the probability objective matter as a property of substance or it can exist subjectively only in the human mind.

3.1 FREQUENCY CONCEPT PROBABILITY

The frequency concept probability defined by Von Mises is as follows. “Suppose that among the N times of experiment, the event A occurred N_A times. The probability P(A) of occurring the event A is defined as the equation:

$$P(A) = \lim_{N \to \infty} \frac{N_A}{N} \;\; .”$$

This equation in the definition means the limiting value of relative frequency. This is the ratio of the constituents of whole events, or of the constituents of mother population. Certainly it is objective. However this numerical value (probability) is for the whole events to the bitter end and is no relation to each event at all. Each experiment result is just if the event A occurred or not. That is, 0 or 1. We should take notice of the fact this definition does not define for each event at all. When we talk about “the reliability on the launch vehicle #5” we shoot only once, this definition certainly gives us difficulty in deeper consideration.

This probability by frequency concept definition can suffice three postulates in the axiom for probability by Kolmogoloff. However, the probability estimated using experimental data does not logically keep the additivity. In other words, we cannot draw a useful conclusion from the probability expression such as “the probability is greater than 0.8 with confidence level of 90%” (Appendix 1).

This definition has been tacitly adopted in the engineering field. Many effective theories have been developed when many samples are available or when the law of large number is preserved or when we predict the property of whole system. However we cannot use this definition when very few samples are available or when we talk about the each event. Then we need to study again other definition for probability.

3.2 AXIOM PROBABILITY

Mathematician Kolmogoloff defined the axiom probability as the abstractive concept. "The probability of event A is the numerical value P(A) which satisfies the following three postulates.

1) P(A) is not negative.
2) The probability of certain event is 1.
3) If A and B are mutually exclusive, then,
   P(A+B) = P(A)+P(B)."

Those three postulates are very natural, acceptable, and apparent conditions for us. If something, whatever it means in the real world, satisfies all of these conditions, we take it as probability and all of the theorems in the probability theory can be applied. Just to make sure, 2) is the condition of normalization and 3) is the necessary condition for additive property.

3.3 CLASSICAL PROBABILITY

It is the definition of probability defined by Laplace that was taken as the classical one by von Mises. This definition says "There are N cases as the result of experiment E and there are Na favorable cases of event A among them. When these cases are equally likely, the probability of event A, P(A) is given by the following equation; $P(A) = \frac{N_A}{N}$."

There have been critics related to the expression "equally likely". It is shown that "no information" is equivalent to the so called the principle of "equal probability" (Appendix 2).

As this probability allude to the property of each experiment this probability can be taken as the special case for the probability as the degree of belief explained in the next section. The special case means when no information is available.

3.4 PROBABILITY AS THE DEGREE OF BELIEF

The certainty on the result of each experiment cannot be the attribute to the experiment but an expression on the state of human mind. The state of human mind is the probability, which shows the degree of belief. Lindley refers the Savage's subjective probability like this [3]. "The probability P(A) is the numerical value assigned to the degree of belief on the truth of the proposition."

For the previous example, the proposition is "Event A occurs in the experiment E." The numerical value is so assigned as to satisfy the three postulates in the axiom of probability by Kolmogoloff.

Needless to say, the probability as degree of belief always attributes to human mind. For example, when we mention the success probability of a pyrotechnic device the probability means the degree of belief on the success of the device and not the attribute of the item such as mass or length of the item. However, we should remind the fact that there are different cases, strong belief of 0.5, weak belief of 0.5, and its intermediate belief of 0.5, for the same probability of 0.5. These situations can be appropriately explained adopting the concept of density of belief and expectation of the density for the probability [4] (Appendix 3).

We would have critics for degree of belief probability if it could be determined scientifically because of its subjective nature. People cannot act always consistently. It is the reason why frequency probability has been adopted in the engineering field.

To avoid this argument, we suppose a human made man called android who acts always consistently. We teach him how to assign a numerical value by the scientific method for the subjective probability as degree of belief. Then he will give us "objective" probability as degree of belief.

The scientific probability calculation procedure we teach android as follows:

(1) When no information is available, the probability is assigned by Laplace's definition of probability. This is a priori probability.
(2) After getting information the probability is calculated as a posteriori probability using Bayes's theorem.

The probability calculated with this procedure can be called "quasi-objective" probability as degree of belief. According to this procedure, we can calculate the probability of belief when only a few data is available for the inspection by attribute; the probability is (r+1)/(n+2), where n is the number of samples and r is the number of success among n [4] (Appendix 3).

Uncertainty in risk can be expressed with the probability as the degree of belief. The probability as degree of belief is also assured for the additive rule, because it satisfies the three postulates of the axiom by Kolmogoloff.

4. CONCEPT OF RISK AND ITS PROPERTY

Risk is the one of most important concepts for human activity. People create, recognize, and acquire values, and behave as not losing them. At the same time people recognize a value people have a fear of not acquiring or of losing it. The reason why they have such a fear is based on the fact that every event is accompanied with uncertainty to some degree on its occurrence. This is the onset of the concept of risk and generally we call risk such a matter of concern.

4.1 DEFINITION OF RISK

The concept of risk is used as a somewhat ambiguous object for the time being. We need to define it exactly for conducting rational risk management.

Risk is always used with the adjective, high or low intending to comparison. Therefore, it is necessary to define risk as a measurable and comparable quantity because risk is always evaluated by comparison with some criteria.

The matter of concern which we aware of risk has always two aspects in the same time. These are amount of value we may lose at the unfavorable event and the probability of occurrence of the event. These are loss and probability of the accident in short.

We cannot compare the multi dimension values as it is. Therefore, the mapping or evaluation function from its dimension to one dimension is needed. The matter of concern has two dimensions, that is, probability and loss. Mathematical multiplication, product, of these is the simplest and meaningful mapping function for our purpose, that is, comparison. If we call this product risk the expression of "high risk" or "low risk" is allowed.

Thus risk is rationally defined as the product of multiplication of the amount of loss if the event occurred and the probability of the occurrence of the event. If the probability is expressed density of the belief, risk is defined as the expectation of loss. This definition of risk gives the foundation of risk concept.

4.2 PROPERTY OF RISK

Risk has the unit of value because loss has the unit of value and because the probability is a dimensionless value. The loss measured with monetary value keeps the additivity and so does the probability as degree of belief. Therefore the risk, which is the product of these, also keeps the additivity. This fact assures the quantitative evaluation for risk.

Value is determined by human kind. The probability as degree of belief also attributes to human kind or android. Therefore risk, which is the product of these, attributes to human kind.

What we should recognize first is not risk but the matter of concern or risky item. Then we evaluate the risk and we would take action(s) if the risk is not low enough for acceptance. Thus we had better to use the words, risk and risky item, discriminately.

In the field of safety, this risky situation is called hazard. Firstly we identify the hazards then evaluate the risk and take action(s) if needed. Safety is the state where risk is low enough to be accepted.

4.3 COMMON LOGARITHM FOR RISK EVALUATION

Based on the above definition of risk, we can evaluate it by multiplying two values, the amount of loss and probability of occurrence. It may not be so difficult to estimate the amount of loss using the unit of value, such as dollar. For most of cases we will be able to estimate the amount of loss within an order error. On the other hand, to give the value to the probability may be much difficult because we are not accustomed to do so. It is hard to recognize the differences intuitively between one in one hundred thousand and one in one-million.

We should remind the probability in the concept of risk is the degree of belief. It should be taken that probability is the measure of expression for the degree of belief. Fortunately it is not necessary to give precise value of probability for evaluating risk but order of the value.

We can evaluate risk absolutely as follows. Suppose the amount of loss, if the event occurs, is 10,000,000 dollars and the probability of the occurrence of the event is 0.01. Then risk is evaluated by multiplying these two values, getting the value of 100,000 dollars.

It is clever to use common logarithm as the risk index rather than risk itself. For the above example, risk index is calculated as 7 – 2 = 5. Thus, risk index 5 means absolute risk value of 100,000 dollars.

5. CONCLUDING REMARKS

Any amount to be compared must be one-dimensional. This may not be necessarily true in the real world. Especially for some emotional words, such as, interesting, happy, sorrowful, etc we may use them with comparative degree without quantitative definition of the terms. However, these are remained in literary expression for the time being, not in the engineering world. If we try to make an android to respond with emotional expression, we will need quantitative definitions and evaluation functions for those words. On the other hand, we have already firm foundation for quantitative risk evaluation.

If we recognize that risk can be measured absolutely using the unit of value, we do not need risk matrix any more for evaluating risk. However it is good practice to keep the two values of loss and probability, to know the character of the risky item.

Based on the above consideration, current documents for risk management need to be somewhat modified including the definition of risk. In addition, it seem to me that we had better to abandon the frequency definition for probability by von Mises in the engineering field, not limited in risk field.

REFERENCE

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[2] MIL-STD-1629A, “Military Standard Procedure for Performing a Failure Mode Effects and Criticality Analysis”, 24 November 1980

[3] D.V. Lindley, “Introduction to Probability and Statistics from a Bayesian Viewpoint”, 1965

[4] N. Hara, “Degree of Belief with a few Data from Inspection by Attribute”, 14th Symposium on Reliability, REAJ, 30 November 2001, (Japanese)

(Appendix 1)

FREQUENCY CONCEPT PROBERBILITY AND ITS ADDITIVITY

Suppose that a system S consists of two independent subsystem A and B, and that the success of S requires the both success of A and of B. That is, reliability block of S consists of series of block A and B.

S: 

Suppose also that we can estimate the reliability of A and B from sample test data as follows. Reliability of A: larger than 0.9 with confidence level of 90 %

Reliability of B: larger than 0.7 with confidence level of 95 %

When we estimate the reliability of A or B from test results available the estimation expression becomes as above, assuming appropriate distribution function and its parameters. In other words, reliability of A is estimated larger than 0.9 but its estimation could be wrong once per ten times. This probability is larger than 0.9 persistently as the property of whole mother population.

Then the success probability of S can be estimated as “larger than 0.63 with the confidence level of 85 %” because both estimations of A and B could be right in chance of 0.9 x 0.95 = 0.85. If S consists of many subsystems and if those reliabilities are estimated with 90 % of confidence level, the confidence level for system S would be very low. For example, “reliability is lager than 0.1 with confidence level of 3 %”. The expression like this is almost nonsense.

From the first, for the estimation of success probability of A the confidence level is selected arbitrary, or following convention from 99%, 95%, 90% or even 60%. Point estimation for the subsystem reliability is useful for the reliability estimation for system. After getting the estimation for reliability as “larger than 0.9 with confidence level of 90 %”, reliability is tacitly assumed as 0.9 ignoring the confidence level for the time being. Or most likely estimate is utilized. These method is almost same as adopting the probability as degree of belief but not logical and no basis for additivity.

(Appendix 2)

EQUALLY LIKELY LAPLACE’S PROBERBILITY

Laplace defined the probability as shown in 3.3 CLASSICAL PROBABILITY. When there is no favorable state A, Laplace’s definition will be rewritten as follows. “If there are N states as the result of experiment E and these N states are equally likely to occur, then the probability of any event Ai is given by the following equation.

$$P(Ai) = \frac{1}{N} \quad .”$$

The a priori provability of 1/N is equally given to every P(A1), P(A2), … , P(An). If we know these may occur equally, each probability is naturally 1/N as the probability of total events is 1.

For the case we do not know these may occur equally but we know there are N states of events, each probability has to be 1/N. If we know the occurrence of any one of states is different from others, this situation is opposite to the condition that “we do not know these may occur equally”. Therefore, no information has to correspond to the equal probability of occurrence.

Usually we assign the probability of 1/6 for the each face of die as we assume the die was made without a trick. Even when we know the die has a trick, if we do not know which face is likely to occur we have to assign same 1/6.

There is a concept of “same” or “identical” in mathematics. However, in the real world “same” means that we do not know the difference. Even if two coins were made by same process in same mint factory, these coins have different lattice defects in microscopic eyes. Much more, tossing force on two coins cannot be same. We have to take “same” if we cannot identify the difference as the limit of human capability.

We can use different coins if we concern about the surface of coin, head or tail. That is, “same” means we do not know the difference on the proposition we concern.

Above consideration suggest us that the state of “equally likely” as the strong belief we have a preconception of “equal” in addition to the state, “we do not know the difference”. We have a preconception of “it must be equal” for the regular die as it is made. Coin is taken with a prejudice that it has tail and head. No one thinks it might be minted with heads only or tails only.

The meaning of “equally likely” in the probability definition by Laplace is that “equally likely” condition with strong belief is very few in the real world. There are rather many “equally likely” conditions with weak belief.

In the first state we do not have information, it is natural we adopt Laplace’s probability as a priori probability. This numerical value due to this definition cannot be a property of substance but an attribute of human mind. It also means degree of belief. Laplace’s definition of probability is included the definition of probability as degree of belief on truth of proposition.

(Appendix 3 – Excerpt from Japanese Manuscript)

DEGREE OF BELIEF WITH A FEW DATA FROM INSPECTION BY ATTRIBUTE

1. PREFACE

We like to know the certainty of occurrence on the event we care, in spite of the number of experiments we can observe. Originally, what the probability means is the numerical value assigned for the subjective idea and is also a matter of mind.

We get firm certainty if we observe many test results, and we get some degree of certainty when we can observe only a few test results. This difference can be expressed in some way. We need the “objective” rule, for deciding the degree of belief.

2. INSPECTION BY ATTRIBUTE

Some kind of tests tells only pass or fail. This kind of test is called inspection by attribute. As most of pyrotechnic devices are consumed by only one time operational test, operational test is destructive test and also inspection by attribute.

Suppose that we’d like to know the success probability of a pyrotechnic device manufactured by the company of which we have no related information such as the credit of manufacturer or past example. We seek the probability as the degree of belief when very few sample of this item are available for inspection by attribute.

The information we input to our android is only the number of samples n, and the number of success items r. We investigate the equation for the probability as the degree of belief he calculate rationally.

3. PROBABILITY AS THE DEGREE OF BELIEF

We adopt the definition of probability as degree of belief. It was established by Savage that degree of belief could be defined as a probability. The probability as degree of belief is no more than the numerical expression for the state of human mind. The probability (p) means “degree of belief for the truth on the proposition” That is, the expression using numerical value from 0 to 1 for the degree of certainty for the truth on the proposition. The value of p can be given any value between 0 and 1. However, the following three values are special cases.

$p \mapsto 1$ ：extremely strong belief of the truth on the proposition. （a symbol, $p \mapsto 1$ means that numerical value 1 is given to p）

$p \mapsto 0$ ：extremely strong belief of the false on the proposition.

$p \mapsto 0.5$ ：entirely no confident about the truth on the proposition, so called, fifty-fifty.

Let us consider the experiment that we pick a stone from urn, which contains white and black stones. The proposition is “the stone picked is white”. It is our problem what is the degree of belief, that is, probability (p), for the truth on the proposition.

If we saw the fact that only white stones are packed into the urn, then $p \mapsto 1$. If only black stones, $p \mapsto 0$. If we knew 30 black and 70 white stones were packed into the urn, then, $p \mapsto 0.7$. If we knew same number of white and black stones are packed, then, $p \mapsto 0.5$. These numerical values are the probability by Laplace’s definition and with strong belief on the value of p.

To the next, suppose the case we do not know how much white and black stones were packed into the urn at all. Even such a case, the stone picked from urn must be white or black. In this case also the probability (p) of truth on the proposition is $p \mapsto 0.5$.

When we knew the same number of white and black stones are packed, it was also same $p \mapsto 0.5$. The difference between these two cases is the contents to p, that is, density of belief on p. The difference is the shape of density of belief on p, $\pi(p)$.

Based on the fact that both case we give $p \mapsto 0.5$, it is concluded as appropriate that “the expectation of the density of the probability is equal to its probability”. This should be taken as a principle accompanied the definition of probability as degree of belief. In other words, we adopt the expectation for converting equation necessary for representing a value for the probability expressed with distribution form. Now, we will have no confusion for writing [=] for [$\mapsto$] instead of writing $p \mapsto E(p)$.

$$p = E(p) = \int_0^1 p\pi(p)dp \qquad \ldots (1)$$

The former is the case we can have strong belief on p because we knew the stones packed. We can express this density of belief distribution with Dirac’s delta function (Fig.1).

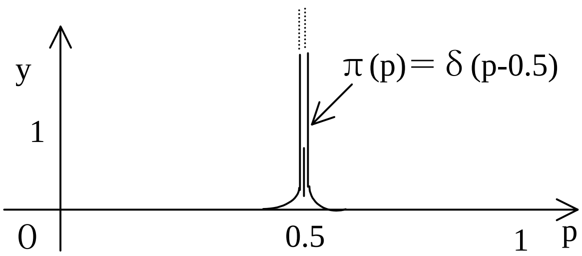

Fig. 1 Strong Density of Belief on p

Conversely, the latter is the case we did not know how stones were packed. It is the weakest degree of belief and we can express the function of density equal distribution from 0 to 1 (Fig. 2).

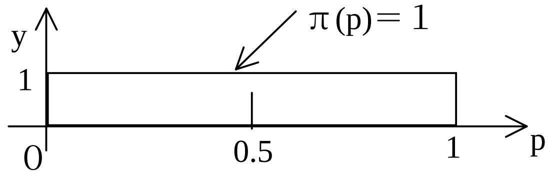

Fig. 2 Weak Density of Belief on p

When we knew same number of white and black stones is packed,

$$\pi(p) = \delta(p-0.5) \qquad \ldots (2)$$

$$\pi(p) = 1 \qquad \ldots (3)$$

Even in the latter case, gradually belief will get stronger by picking up stones one by one. That is, by seeing data the shape of density of belief will be deformed.

Density of belief $\pi(p)$ has the following natures.

$$\pi(p) = 0, \quad p < 0, \quad p > 1$$

$$\pi(p) \geq 0, \quad 0 \leq p \leq 1 \qquad \ldots (4)$$

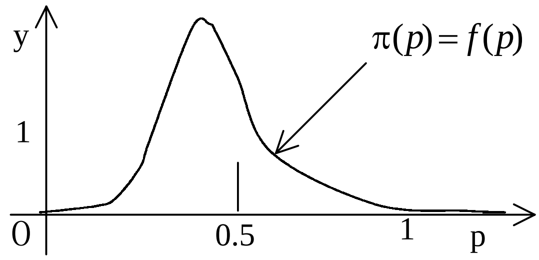

Fig. 3 General Density of Belief on p

In addition, we can add the equation as the condition for normalization.

$$\int_{-\infty}^{\infty} \pi(p)dp = \int_0^1 \pi(p)dp = 1 \qquad \ldots (5)$$

Fig. 3 shows the general density of belief on p.

4. PROBABILITY BY LAPLACE AND D'ALENBERT

Let us consider the experiment of picking out two stones independently from the previous urn. Letting p for the probability of white stone from urn, the probability of both white is p^2. If the density of belief on p is expressed as $\pi(p)$, then the equation (1) turns to the equation (6).

$$E(W,W) = {}_2C_2 \int_0^1 p^2 \pi(p) dp \quad \ldots (6)$$

Similarly, in case of one is white and the other is black and in case of both black the probability is expressed by the equations (7) and (8) respectively.

$$E(W,B) = {}_2C_1 \int_0^1 p(1-P) \pi(p) dp \quad \ldots (7)$$

$$E(B,B) = {}_2C_0 \int_0^1 (1-P)^2 \pi(p) dp \quad \ldots (8)$$

If we enter equation (2) for π (p) in (6), (7), and (8) and if calculate them, we get values of 1/4、1/2, 1/4 respectively. This corresponds to the Binomial distribution B(2,0.5).

On the other hand, entering equation (3) to π (p), we get values of 1/3、1/3、1/3 respectively. This result shows equal distribution of 1/3 to the possible three states.

Tossing two coins simultaneously, the probabilities of both heads, one is head and the other is tail, and both tail, were determined as 1/3,1/3, and 1/3 by D'Alenbert. It is reported that Laplace who was a D'Alenbert's pupil corrected them as 1/4, 1/2, and 1/4 respectively. Each coin has head and tail. Therefore this is the case of strong belief on p=0.5. In the real experiment, the statistics will show the nearly Laplace's probability.

Besides, in the case of picking out n coins independently from urn we can conduct same calculation. Final result will be Binomial distribution B(n,0.5) or equal distribution depending on π (p).

5. DEGREE OF BELIEF AFTER GETTING DATA

If degree of belief is strong enough as shown with Dirac's delta function, the probability is unchangeable by seeing finite number of data. However, if a priori belief is weak, the degree of belief, that is probability, will change by seeing data. This change can be calculated using Bayes's Theorem.

Bayes's Theorem asserts the following. "A posteriori density after observing data is proportional to the product of likelihood of data and a priori density."

The probability p of after observing data is the expectation of a posteriori density on p.

Inspection by attribute is a test which provides data of only success or fail. If we take success for white stone, it corresponds to the experiment of picking out stones from urn of the case we did not see the packing of stones. All of stones may be white or fairy number of black stones might be mixed.

Firstly, for the initial state it is appropriate by the previous consideration to adopt equation (3) for a priori density of belief.

Data **X** (x1, x2, …, xn) means a series of success, and fail. As these are independent, the order is no relation to the probability but only the number of test samples, n, and the number of success r affects a posteriori density.

After observing data **X**, a posteriori density, π (p|**X**) ,

$$\pi(p|X) \propto L(X|p)\pi(p) \quad \ldots (9)$$

Where, L(**X**|p) is the likelihood of data **X** on p.

The likelihood of data, r success among n samples, is $p^r(1-p)^{n-r}$. Then,

$$\pi(p \mid X) \propto p^r (1-p)^{n-r} \times 1 \quad \ldots (10)$$

From the condition of equation (5), we can determine the constant utilizing Beta integral formula.

$$\pi(p \mid X) = \frac{\Gamma(n+2)}{\Gamma(r+1)\Gamma(n-r+1)} p^r (1-p)^{n-r} \quad \ldots (11)$$

Equation (11) is no more than a β -distribution. Therefore, the degree of belief after observing data, that is, probability p is given by the expectation of equation (1).

$$p = E(\pi(p \mid X)) = \frac{r+1}{n+2} \quad \ldots (12)$$

Where, n is number of test sample, r is number of success among n. Equation (12) is the probability we seek after observing data. This is known as Laplace's rule of succession.

By this equation, from the initial state of reliability 0.5, it rises to 0.9 if we observe the consecutive 8 successes without failure. If we observe 18 of successes without failure it will be but only 0.95. If a failure is observed the reliability will not rise easily anymore. This explains we need the research and counter measure if we experience a failure.

6. CLOSING REMARKS

This manuscript is the review of my old presentation titled "Reliability due to Inspection by Attribute" at the internal symposium of NASDA in 1978.